Reconstructing the two right-handed neutrino model

Alejandro Ibarra

Instituto de Física Teórica, CSIC/UAM, C-XVI, Universidad Autónoma de Madrid
Cantoblanco, 28049 Madrid, Spain
E-mail: `alejandro.ibarra@cern.ch`

ABSTRACT: In this paper we propose a low-energy parametrization of the two right-handed neutrino model, and discuss the prospects to determine experimentally these parameters in supersymmetric scenarios. In addition, we present exact formulas to reconstruct the high-energy leptonic superpotential in terms of the low-energy observables. We also discuss limits of the three right-handed neutrino model where this procedure applies.

KEYWORDS: Supersymmetric Standard Model, Beyond Standard Model, Neutrino Physics.

Contents

1. Introduction

The observed neutrino masses can be naturally accounted for by adding to the Standard Model Lagrangian three heavy singlets, usually identified with the right-handed neutrinos. This framework is denoted as the type I see-saw mechanism, or simply the see-saw mechanism [1]. Furthermore, this framework can also accommodate, although not explain, the large atmospheric and solar mixing angles, as well as the small 13 element in the leptonic mixing matrix.

Although the see-saw mechanism describes qualitatively well the observations, it lacks predictive power. The reason is that the leptonic Lagrangian is defined at high energies by 21 parameters, whereas experimentally we can only measure at most 12 parameters. The 9 remaining parameters are lost in the decoupling process and can be arbitrarily chosen, without leaving any other trace at low energies.[1]

In the supersymmetric version of the see-saw mechanism the neutrino Yukawa coupling affects the renormalization group evolution of the slepton parameters above the decoupling scale [3], and thus leave an imprint that could be disentangled using low energy experiments (after making some assumptions on the structure of the slepton parameters at the cut-off

---

[1]There is additional information encoded in dimension six operators, but unfortunately they are too suppressed to be observed experimentally [2].

scale). For instance, phenomena such as rare decays [4–6] electric dipole moments [7], or mass splittings among the different generations of charged sleptons or sneutrinos [8] provide additional information about the see-saw parameters.

This information is encoded in a 3×3 hermitian matrix, $P = \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$, that depends on nine parameters (six moduli and three phases). It can be shown that these nine parameters are precisely the complementary information needed to reconstruct the high energy Lagrangian [9, 10]. The positive consequence of this observation is that the see-saw mechanism can be parametrized just in terms of low energy observables; the negative consequence is that for any set of low energy lepton and slepton parameters, there is a high energy theory with three right-handed neutrinos that can accommodate it, and in consequence, the see-saw model cannot be disproved.

In this parametrization, the correspondence between high energy and low energy parameters is one to one. Therefore, any additional hypothesis on the high-energy see-saw parameters would lead to predictions on the low energy parameters. Some well motivated assumptions have been proposed in the literature, such as texture zeros, symmetric matrices or the two right-handed neutrino model, and their consequences studied in a number of papers. In this work we will concentrate on the latter possibility [11, 12]. The motivation is the following. Leaving aside the LSND anomaly, experiments have measured two mass splittings, indicating that at least two new scales have to be introduced. The two new scales could correspond to the masses of two right handed neutrinos, being the third one not necessary to reproduce the oscillation experiments. Therefore, a model with just two right handed neutrinos could explain all the observations, but it depends on less parameters.

The two right-handed neutrino model has some interesting features. Namely, the two right-handed neutrino model predicts a hierarchical spectrum for the light neutrinos, being the lightest strictly massless. Furthermore, since there are only two non-vanishing masses, there is only one Majorana phase, corresponding to the phase difference between these two eigenvalues. Analogously to the model with three right-handed neutrinos, there are three mixing angles and one Dirac phase.

Since the number of parameters involved is smaller, we expect predictions not only for the neutrino mass matrix, but also for the low-energy slepton mass matrix (under some assumptions about the structure of this matrix at high energy), as well as a simpler reconstruction procedure of the high energy parameters. In section 2 we will show that in the two right-handed neutrino model some relations arise among different elements in the slepton mass matrix. We will also present a possible parametrization of this model just in terms of low energy observables. In section 3 we will discuss the prospects to determine experimentally these parameters, and accordingly the feasibility of the reconstruction of the high-energy parameters. In section 4 we will propose an exact reconstruction procedure, and illustrate it for a particular possibility of the low energy parameters. In section 5 we will analyze different limits of the three right handed neutrino model that can be well described in practice by a two right-handed neutrino model, and where the reconstruction procedure proposed in this paper applies. Finally, in section 6 we will present our conclusions. We will also present an appendix with a more elaborated reconstruction procedure.

2. Parametrizations of the two right-handed neutrino model

In the appropriate basis, the two right-handed neutrino (2RHN) model is defined at high energies by a 2×3 Yukawa matrix and two right-handed neutrino masses, M_1 and M_2. This amounts to eight moduli and three phases. On the other hand, at low energies the neutrino mass matrix is defined by five moduli (two masses and three mixing angles) plus two phases (the Dirac phase and the Majorana phase). Therefore, the number of unknown parameters is reduced to three moduli and one phase.

Using the parametrization presented in [5], the neutrino Yukawa coupling can be expressed as:

$$\mathbf{Y}_\nu = D_{\sqrt{\mathbf{M}_\nu}} R D_{\sqrt{m}} U^\dagger / \langle H_u^0 \rangle, \tag{2.1}$$

where $D_{\sqrt{\mathbf{M}_\nu}} = \text{diag}(\sqrt{M_1}, \sqrt{M_2})$ is the diagonal matrix of the square roots of the right-handed masses, $D_{\sqrt{m}} = \text{diag}(0, \sqrt{m_2}, \sqrt{m_3})$ is the diagonal matrix of the squared roots of the physical masses of the light neutrinos, $\langle H_u^0 \rangle$ is the vacuum expectation value of the neutral component of the up-type Higgs doublet, U is the leptonic mixing matrix [13], and R is a 2×3 complex matrix, which parametrizes the information that is lost in the decoupling of the right-handed neutrinos. It is possible to prove that R has the following structure [14, 15]

$$R = \begin{pmatrix} 0 & \cos z & \xi \sin z \\ 0 & -\sin z & \xi \cos z \end{pmatrix}, \tag{2.2}$$

where z is a complex parameter and $\xi = \pm 1$ is a parameter that accounts for a discrete indeterminacy in the Yukawa coupling.

Notice that we have included all the low energy phases in the definition of the matrix U, i.e. we have written the leptonic mixing matrix in the form $U = V \, \text{diag}\,(1, e^{-i\phi/2}, 1)$, where ϕ is the Majorana phase and V has the form of the CKM matrix:

$$V = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{2.3}$$

so that the neutrino mass matrix is $\mathcal{M} = U^* \text{diag}(0, m_2, m_3) U^\dagger$. It is straightforward to check that eq. (2.1) indeed satisfies $\mathcal{M} = \mathbf{Y}_\nu^T \text{diag}(M_1^{-1}, M_2^{-1}) \mathbf{Y}_\nu \langle H_u^0 \rangle^2$.

The Yukawa coupling affects the renormalization group equation of the slepton parameters through the combination $P = \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$, that depends in general on six moduli and three phases. Since the Yukawa coupling depends in the 2RHN model on only three unknown moduli and one phase, so does P, and consequently it is possible to obtain predictions on the moduli of three P-matrix elements and the phases of two P-matrix elements. Namely, from eq. (2.1) one obtains that:

$$U^\dagger P U = U^\dagger \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu U = D_{\sqrt{m}} R^\dagger D_{\mathbf{M}_\nu} R D_{\sqrt{m}} / \langle H^0 \rangle^2. \tag{2.4}$$

Since $m_1 = 0$, it follows that $(U^\dagger P U)_{1i} = 0$, for $i = 1, 2, 3$, leading to three relations among

the elements in P. For instance, one could derive the diagonal elements in P in terms of the off-diagonal elements:

$$P_{11} = -\frac{P_{12}^* U_{21}^* + P_{13}^* U_{31}^*}{U_{11}^*},$$
$$P_{22} = -\frac{P_{12} U_{11}^* + P_{23}^* U_{31}^*}{U_{21}^*},$$
$$P_{33} = -\frac{P_{13} U_{11}^* + P_{23} U_{21}^*}{U_{31}^*}. \tag{2.5}$$

The observation of these correlations would be non-trivial tests of the 2RHN model.

The relations for the phases arise from the hermicity of P, since the diagonal elements in P have to be real. Taking as the independent phase the argument of P_{12}, one can derive from eq. (2.5) the arguments of the remaining elements:

$$e^{i \arg P_{13}} = \frac{-i \,\mathrm{Im}(P_{12} U_{21} U_{11}^*) \pm \sqrt{|P_{13}|^2 |U_{11}|^2 |U_{31}|^2 - [\mathrm{Im}(P_{12} U_{21} U_{11}^*)]^2}}{|P_{13}| U_{31} U_{11}^*},$$
$$e^{i \arg P_{23}} = \frac{i \,\mathrm{Im}(P_{12} U_{21} U_{11}^*) \pm \sqrt{|P_{23}|^2 |U_{21}|^2 |U_{31}|^2 - [\mathrm{Im}(P_{12} U_{21} U_{11}^*)]^2}}{|P_{23}| U_{31} U_{21}^*}, \tag{2.6}$$

where the $\pm$ sign has to be chosen so that the eigenvalues of P are positive. It is important to remark that the hermicity of P is not guaranteed for any value of P_{12}, $|P_{13}|$, $|P_{23}|$; only some particular ranges for the parameters are allowed, corresponding to the values for which the arguments of the square roots in eq. (2.6) are positive.

We conclude then that the P-matrix parameters P_{12}, $|P_{13}|$ and $|P_{23}|$ can be regarded as independent and can be used as an alternative parametrization of the 2RHN model. Together with the five moduli and the two phases of the neutrino mass matrix, sum up to the eight moduli and the three phases necessary to reconstruct the high-energy Lagrangian of the 2RHN model.

3. Can we reconstruct (realistically) the complete theory from low energy observables?

An interesting feature of the 2RHN model is that it could be feasible to reconstruct the neutrino mass matrix. This model predicts that one of the neutrino masses vanishes, and in consequence the spectrum is necessarily hierarchical. In this scenario, the masses would have been already determined by present oscillation experiments: $m_2 = \sqrt{\Delta m_{21}^2}$ and $m_3 = \sqrt{\Delta m_{31}^2}$, where $\Delta m_{21}^2 = (7.1-8.9) \times 10^{-5}$ eV and $\Delta m_{31}^2 = (1.4-3.3) \times 10^{-3}$ eV are the 3σ ranges for the solar and atmospheric mass splittings obtained from the combined analysis of global data [16]. In the next few years, the measurements of the mass splittings are expected to improve. To be specific, the error in the measurement of the atmospheric mass splitting, Δm_{31}^2, is expected to be reduced by experiments using the CERN to Gran Sasso neutrino beam (ICARUS [17] and OPERA [18]), MINOS [19], NOνA [20] and particularly

T2K [21], that will probably reduce the present error by one order of magnitude. There are also proposals to reduce the error in the measurement of the solar mass splitting, Δm_{21}^2. Namely, if the SuperK detector was loaded with gadolinium, it would be possible to reduce the error by a factor of six [23].

Concerning the mixing angles, two of them have been determined to a good accuracy by present experiments: $\sin^2 \theta_{12} = 0.24 - 0.40$ and $\sin^2 \theta_{23} = 0.34 - 0.68$ at the 3σ level [16]. The T2K experiment will probably reduce the error in $\sin^2 \theta_{23}$ by a factor of two. On the other hand, the error in $\sin^2 \theta_{12}$ will not be substantially reduced in the near future, although an experiment similar to KamLAND but with a baseline slightly shorter, $L \sim$ 60km, could reduce the error by a factor of four [24].

The angle θ_{13} has not been detected yet but the global analysis sets the upper bound $\sin^2 \theta_{13} < 0.046$, also at 3σ [16]. Ongoing experiments such as MINOS, ICARUS or OPERA could improve the present limit by a factor of two, while future experiments as D-Chooz [22] by a factor of four, and T2K or NOνA by a factor of ten. If θ_{13} is large, the combined analysis of the experiments could provide some information about the Dirac phase δ. However, the detailed analysis of CP violation in the neutrino sector will require superbeams. For instance, improving the proton intensity at JHF to 4MW and using the proposed Hyper-Kamiokande detector, it could be possible to reach a sensitivity below 3×10^{-4} for $\sin^2 \theta_{13}$ and around $10° - 20°$ for δ [21].

Thus we find that there are good prospects to determine the masses, mixing angles and δ to the percent level in the next 10-20 years [25]. However, the measurement of the Majorana phase is still very challenging. Although it could in principle be measured in experiments of neutrinoless double beta decay, the sensitivity of the projected experiments is still far from observing this process in the case of a hierarchical spectrum of neutrinos [26].

Concerning the matrix P, there are also very good prospects to determine the relevant parameters: P_{12}, $|P_{13}|$ and $|P_{23}|$. With experiments of rare decays, it would be possible to determine the moduli of the off-diagonal elements of P. The present bounds on the rates for these decays are $BR(\mu \to e\gamma) \lesssim 1.2 \times 10^{-11}$ [27], $BR(\tau \to \mu\gamma) < 3.1 \times 10^{-7}$ [28] (Belle) or $< 6.8 \times 10^{-8}$ [29] (BaBar), and $BR(\tau \to e\gamma) < 3.9 \times 10^{-7}$ [30], that translate into $|P_{12}| < 3 \times 10^{-4}$, $|P_{23}| < 0.09$ and $|P_{13}| < 0.09$, for $\tan\beta = 10$ and typical slepton masses of ~ 400GeV [31].

In the next few years, the sensitivity of the experiments to these processes is also expected to improve. The MEG experiment expects to reach a sensitivity of $BR(\mu \to e\gamma) \lesssim 10^{-13}$ [32], that would translate into $|P_{12}| \lesssim 3 \times 10^{-5}$. Although presently the most stringent constraints on $|P_{12}|$ arise from the process $\mu \to e\gamma$, this role could be played in the future by experiments on the coherent conversion of muons into electrons in nuclei.[2] The current experimental bounds are $R(\mu^- \text{Ti} \to e^- \text{Ti}) < 6.1 \times 10^{-13}$ [33] and $R(\mu^- \text{Au} \to e^- \text{Au}) < 5 \times 10^{-13}$ [34], and are expected to be improved by the PRISM/PRIME experiment, aiming to a sensitivity of 10^{-18} [35], or the CERN neutrino factory, aiming to 10^{-19} [36].

---

[2] When the photon penguin diagram dominates the contribution in the $\mu - e$ conversion, the conversion rate is approximately three orders of magnitude smaller than the branching ratio for $\mu \to e\gamma$.

On the other hand, B-factories are also τ-factories, and constitute splendid opportunities to search for rare τ decays. Future super B-factories could produce of the order of 10^{10} τ pairs at a luminosity of 10 ab^{-1}, allowing to probe branching ratios for the rare τ decays down to the level of $10^{-8} - 10^{-9}$ [37].

The only phase that appears in P could be determined from measurements of the electric dipole moment of the electron, whose present bound is $d_e < 1.6 \times 10^{-27}$ e cm [38]. In order to disentangle the contribution from the neutrino Yukawa couplings it would be necessary to determine the phases in the neutralino or the chargino sector from other experiments, most probably the LHC or the ILC. The prospects to improve the sensitivity of the experiments to detect electron dipole moments are also very encouraging. By using the metastable $a(1)[^3\Sigma^*]$ state of PbO it could be possible to improve the sensitivity of the experiments to $d_e \lesssim 10^{-29}$ e cm [39], or even to $d_e \lesssim 10^{-31}$ e cm in a few years [40]. In the longer term, it could be possible to improve the sensitivity to $d_e \lesssim 10^{-35}$ e cm using solid state techniques [41]. Incidentally, it has been argued that the effect of the phases in the neutrino Yukawa coupling would not be observed in the electron dipole moment before this sensitivity is reached [43].

In summary, the prospects to detect or further constrain the low-energy parameters of the 2RHN model are very encouraging, and accordingly the prospects to reconstruct the high energy theory from experiments. It is remarkable that most of the problems encountered when determining the low energy parameters of the 3RHN model disappear in the 2RHN model. Namely, in the 3RHN model there are two Majorana phases, and despite one combination of them could be measured in experiments of neutrinoless double beta decay, there is no proposed experimental set-up to measure the second combination. Furthermore, in the matrix P there are more independent parameters in the 3RHN model than in the 2RHN model, and the prospects to measure them are not so encouraging. In particular, the diagonal elements in P would be quite hard to measure, although it could be possible to measure the largest mass difference among the sleptons or the sneutrinos in colliders [8]. On the other hand, the measurement of the smallest mass difference seems to be far out of the reach of the proposed future experiments. Concerning the electric dipole moments, there are some prospects to improve the present bound on the muon electric dipole moment, $d_\mu < 7 \times 10^{-19}$ e cm [44], to $d_\mu \lesssim 10^{-24}$ e cm using the muon ring at BNL [45] or even to $d_\mu \lesssim 10^{-26}$ e cm at the neutrino factory [36]. On the other hand, the present bounds on the τ electric dipole moment are very loose, $-2.2 < \mathrm{Re}(d_\tau) < 4.5(\times 10^{-17})$ e cm [46], and there are no prospects to improve them substantially in the near future.

It was pointed out in [47, 48, 15] that the phase of z is the only phase that plays a role in the mechanism of leptogenesis [49]. Therefore, the see-saw mechanism could be parametrized in terms of the leptogenesis phase instead of the phase that induces electric dipole moments. Nevertheless, the leptogenesis mechanism depends on assumptions that are harder to test than the assumptions needed to disentangle the phase in P from electric dipole moments, and thus this possibility does not seem to be very practical. In any case, we find very remarkable the close relation between electric dipole moments and leptogenesis in the 2RHN model.

4. The reconstruction procedure

In section 2 we have discussed that the complete Lagrangian can be written in terms of the five moduli and two phases of the neutrino mass matrix, and the three independent moduli and two phases of the matrix P, that is involved in the radiative corrections of the slepton parameters. In this section we will derive *exact* formulas for the high energy parameters in terms of these low energy parameters [50]

To this end, we will use the parametrization of the Yukawa couplings in eq. (2.1), so that all our ignorance of the high energy theory is encoded in the right-handed neutrino masses, M_1 and M_2, and the complex parameter in the matrix R, z. Let us define the hermitian matrix $Q \equiv U^\dagger P U$, that depends *exclusively* on parameters that in principle could be measured in low energy experiments. The first row and column vanish and yield the relations among the P-matrix elements already presented in eq. (2.5). On the other hand, the remaining elements Q_{22}, Q_{23}, Q_{33}, can be written in terms of the high-energy parameters M_1, M_2 and z. Therefore, one can invert the equations to derive exact expressions for the high-energy parameters in terms of the low energy parameters in Q. These expressions are:

$$M_1 = \frac{1}{2}\left[\sqrt{\left(\frac{Q_{33}}{m_3} + \frac{Q_{22}}{m_2}\right)^2 + \frac{(Q_{23} - Q_{23}^*)^2}{m_2 m_3}} - \sqrt{\left(\frac{Q_{33}}{m_3} - \frac{Q_{22}}{m_2}\right)^2 + \frac{(Q_{23} + Q_{23}^*)^2}{m_2 m_3}}\right]\langle H_u^0\rangle^2,$$

$$M_2 = \frac{1}{2}\left[\sqrt{\left(\frac{Q_{33}}{m_3} + \frac{Q_{22}}{m_2}\right)^2 + \frac{(Q_{23} - Q_{23}^*)^2}{m_2 m_3}} + \sqrt{\left(\frac{Q_{33}}{m_3} - \frac{Q_{22}}{m_2}\right)^2 + \frac{(Q_{23} + Q_{23}^*)^2}{m_2 m_3}}\right]\langle H_u^0\rangle^2,$$

$$\cos 2z = \left(\frac{Q_{33}^2}{m_3^2} - \frac{Q_{22}^2}{m_2^2} + \frac{(Q_{23} + Q_{23}^*)(Q_{23} - Q_{23}^*)}{m_2 m_3}\right)\frac{\langle H_u^0\rangle^4}{M_2^2 - M_1^2}. \tag{4.1}$$

To complete the reconstruction procedure, the Yukawa coupling would be derived from these parameters using eq. (2.1) and where the discrete parameter ξ in eq. (2.2) is determined by:

$$\xi = \frac{\sqrt{m_2 m_3}}{Q_{23}\langle H_u^0\rangle^2}(M_1 \sin z \cos z^* - M_2 \cos z \sin z^*). \tag{4.2}$$

In the case that all the parameters are real, the expressions greatly simplify:

$$M_1 = \frac{1}{2}\left[\frac{Q_{33}}{m_3} + \frac{Q_{22}}{m_2} - \sqrt{\left(\frac{Q_{33}}{m_3} - \frac{Q_{22}}{m_2}\right)^2 + \frac{4Q_{23}^2}{m_2 m_3}}\right]\langle H_u^0\rangle^2,$$

$$M_2 = \frac{1}{2}\left[\frac{Q_{33}}{m_3} + \frac{Q_{22}}{m_2} + \sqrt{\left(\frac{Q_{33}}{m_3} - \frac{Q_{22}}{m_2}\right)^2 + \frac{4Q_{23}^2}{m_2 m_3}}\right]\langle H_u^0\rangle^2,$$

$$\cos 2z = \left(\frac{Q_{33}^2}{m_3^2} - \frac{Q_{22}^2}{m_2^2}\right)\frac{\langle H_u^0\rangle^4}{M_2^2 - M_1^2}. \tag{4.3}$$

We would like to illustrate now the reconstruction procedure for an interesting possibility for the matrix P. In the previous section we argued that among all the elements in

P, the ones with better prospects to be constrained or measured were the off-diagonal ones. Therefore, it will prove convenient from the phenomenological point of view to use P_{12}, $|P_{13}|$ and $|P_{23}|$, together with the neutrino mass matrix, to parametrize the 2RHN model. Furthermore, the stringent constraints on the process $\mu \to e\gamma$ suggests us to take the limit $|P_{12}| \ll |P_{13}|, |P_{23}|$.[3] This limit yields a very constrained structure for the P-matrix,

$$P \simeq \begin{pmatrix} -P_{13}^* \frac{U_{31}^*}{U_{11}^*} & 0 & P_{13} \\ 0 & -P_{23}^* \frac{U_{31}^*}{U_{21}^*} & P_{23} \\ P_{13}^* & P_{23}^* & -P_{13} \frac{U_{11}^*}{U_{31}^*} - P_{23} \frac{U_{21}^*}{U_{31}^*} \end{pmatrix}, \qquad (4.4)$$

that in turn would imply predictions for the mass splittings among the sleptons in terms of the branching ratios for the processes $\tau \to \mu\gamma$ and $\tau \to e\gamma$. Note also that the phases in the matrix P are fixed in terms of the phases in the leptonic mixing matrix. Namely, to make the diagonal elements of the matrix P real, as required by hermicity, one has to require

$$\arg P_{13} \simeq \arg U_{11} - \arg U_{31} + \pi,$$
$$\arg P_{23} \simeq \arg U_{21} - \arg U_{31} + \pi. \qquad (4.5)$$

We have also resolved the $\pm$ ambiguity in the expressions for the arguments of P_{13} and P_{23} in eq. (2.6) by applying the Sylvester criterion, in order to yield positive eigenvalues in P. Note that the Majorana phase, ϕ, will not appear in these expressions, and the only phase that will appear is the Dirac phase, suppressed by the small value of θ_{13}. To be precise, if we substitute $\theta_{23} \simeq \pi/4$, $\theta_{12} \simeq \pi/6$ and we take into account that θ_{13} is small, we obtain

$$P_{13} \simeq -|P_{13}|(1 + i\,\sqrt{3}\sin\delta\sin\theta_{13}),$$
$$P_{23} \simeq |P_{23}|(1 + 2i\,\sqrt{3}\sin\delta\sin\theta_{13}). \qquad (4.6)$$

Therefore, in the limit where $|P_{12}| \ll |P_{13}|, |P_{23}|$, all the elements in P are expected to be real to a good approximation, and accordingly the contribution to the electric dipole moments from the slepton parameters is expected to be very small. Note also that requiring positive eigenvalues for P requires that P_{13} is negative and P_{23} positive.

The high-energy parameters can be easily reconstructed from the general formulas in eq. (4.1), where the relevant elements in the matrix Q read:

$$Q_{22} \simeq |P_{13}| \frac{|U_{23}|^2}{|U_{11}||U_{31}|} + |P_{23}| \frac{|U_{13}|^2}{|U_{21}||U_{31}|},$$
$$Q_{33} \simeq |P_{13}| \frac{|U_{22}|^2}{|U_{11}||U_{31}|} + |P_{23}| \frac{|U_{12}|^2}{|U_{21}||U_{31}|},$$
$$Q_{23} \simeq -|P_{13}| \frac{U_{22}^* U_{23}}{|U_{11}||U_{31}|} - |P_{23}| \frac{U_{12}^* U_{13}}{|U_{21}||U_{31}|}. \qquad (4.7)$$

---

[3]It is important to stress that there is no solid experimental evidence supporting this possibility. The only reason why the bound on $\mu \to e\gamma$ is stronger than the bounds on $\tau \to (\mu, e)\gamma$ is that presently the muon sources are more intense than the tau sources. Other possibilities are a priori equally plausible from the phenomenological point of view, despite there is a theoretical prejudice in favor of $|P_{12}| \ll |P_{13}|, |P_{23}|$. The analysis for other scenarios will be presented elsewhere.

To show the analytical results, we will limit ourselves to the case where $\theta_{23} \simeq \pi/4$, $\theta_{12} \simeq \pi/6$, and $\theta_{13} \simeq 0$, as suggested by data. Depending on the values of the remaining non-vanishing parameters, we can distinguish two limits: $|P_{13}| \ll |P_{23}|$ and $|P_{23}| \ll |P_{13}|$. The structure of the P-matrix is different in each case, and reads:

$$P \simeq |P_{23}| \begin{pmatrix} \lambda/\sqrt{6} & 0 & -\lambda \\ 0 & 1 & 1 \\ -\lambda & 1 & 1 \end{pmatrix}, \qquad \text{for } \lambda = \frac{|P_{13}|}{|P_{23}|} \ll 1, \tag{4.8}$$

$$P \simeq |P_{13}| \begin{pmatrix} 1/\sqrt{6} & 0 & -1 \\ 0 & \lambda & \lambda \\ -1 & \lambda & \sqrt{6} \end{pmatrix}, \qquad \text{for } \lambda = \frac{|P_{23}|}{|P_{13}|} \ll 1. \tag{4.9}$$

In this approximation there are no phases in P, although there could be phases in the neutrino mass matrix. Let us analyze first the case where the neutrino mass matrix is also real, and later on, the general case allowing complex parameters.

4.1 Real case

The reconstruction of the high-energy parameters in terms of the low energy parameters is straightforward using eq. (4.3). The reconstructed high energy parameters in each limit read:

- $|P_{13}| \ll |P_{23}|$

$$M_1 \simeq 2\sqrt{\frac{2}{3}} \frac{|P_{13}|}{m_2} \langle H_u^0 \rangle^2,$$
$$M_2 \simeq \frac{2|P_{23}|}{m_3} \langle H_u^0 \rangle^2,$$
$$\cos 2z \simeq 1. \tag{4.10}$$

so that the reconstructed Yukawa coupling is:

$$\mathbf{Y}_\nu \simeq \sqrt{|P_{23}|} \begin{pmatrix} \sqrt{\frac{|P_{13}|}{\sqrt{6}|P_{23}|}} & \sqrt{\sqrt{\frac{3}{8}}\frac{|P_{13}|}{|P_{23}|}} & -\sqrt{\sqrt{\frac{3}{8}}\frac{|P_{13}|}{|P_{23}|}} \\ -\frac{|P_{13}|}{2|P_{23}|} & 1 & 1 \end{pmatrix}. \tag{4.11}$$

It is interesting that in this limit the lightest right-handed mass is essentially determined by the rate for the process $\tau \to e\gamma$, while the heaviest one, by the process $\tau \to \mu\gamma$. On the other hand, the complex angle in R is such that in this case $\cos 2z$ is very close to one, independently of the values of $|P_{13}|$ and $|P_{23}|$, as long as $|P_{13}| \ll |P_{23}|$.

- $|P_{23}| \ll |P_{13}|$

$$M_1 \simeq \frac{8|P_{23}|}{3m_2 + 4m_3} \langle H_u^0 \rangle^2,$$
$$M_2 \simeq \frac{(3m_2 + 4m_3)|P_{13}|}{\sqrt{6}m_2 m_3} \langle H_u^0 \rangle^2,$$
$$\cos 2z \simeq \frac{3m_2 - 4m_3}{3m_2 + 4m_3}, \tag{4.12}$$

for the case in which the light neutrinos have the same CP parities, i.e. when $\phi = 0$, and

$$M_1 \simeq \frac{8|P_{23}|}{-3m_2 + 4m_3}\langle H_u^0 \rangle^2,$$

$$M_2 \simeq \frac{(-3m_2 + 4m_3)|P_{13}|}{\sqrt{6}m_2 m_3}\langle H_u^0 \rangle^2,$$

$$\cos 2z \simeq \frac{3m_2 + 4m_3}{3m_2 - 4m_3}, \tag{4.13}$$

when they have opposite CP parities, $\phi = \pi$.

The reconstructed Yukawa coupling is in this case:

$$\mathbf{Y}_\nu \simeq \sqrt{\sqrt{6}|P_{13}|}\left(\begin{array}{ccc} \sqrt{\frac{\sqrt{6}|P_{23}|}{|P_{13}|}}\frac{m_2}{3m_2 + 4m_3} & \sqrt{\frac{|P_{23}|}{\sqrt{6}|P_{13}|}} & \sqrt{\frac{|P_{23}|}{\sqrt{6}|P_{13}|}}\frac{-3m_2 + 4m_3}{3m_2 + 4m_3} \\ -\frac{1}{\sqrt{6}} & \frac{\sqrt{6}m_2}{3m_2 + 4m_3}\frac{|P_{23}|}{|P_{13}|} & 1 \end{array} \right), \tag{4.14}$$

when $\phi = 0$ and

$$\mathbf{Y}_\nu \simeq \sqrt{\sqrt{6}|P_{13}|}\left(\begin{array}{ccc} -\sqrt{\frac{\sqrt{6}|P_{23}|}{|P_{13}|}}\frac{m_2}{-3m_2 + 4m_3} & \sqrt{\frac{|P_{23}|}{\sqrt{6}|P_{13}|}} & \sqrt{\frac{|P_{23}|}{\sqrt{6}|P_{13}|}}\frac{3m_2 + 4m_3}{-3m_2 + 4m_3} \\ -\frac{i}{\sqrt{6}} & \frac{-i\sqrt{6}m_2}{-3m_2 + 4m_3}\frac{|P_{23}|}{|P_{13}|} & i \end{array} \right), \tag{4.15}$$

when $\phi = \pi$.

In the case with $|P_{23}| \ll |P_{13}|$, the behaviour is opposite to the previous one: the lightest right-handed mass is determined by $\tau \to \mu\gamma$, and the heaviest by $\tau \to e\gamma$. In this limit, $\cos 2z$ is also independent of $|P_{13}|$ and $|P_{23}|$, and takes a negative value, $\cos 2z \simeq -0.75$, for $\phi = 0$ and positive, $\cos 2z \simeq -1.33$, for $\phi = \pi$.

The numerical results for the case with $\phi = 0$ are shown in figure 1 for different values of $|P_{13}|$ and $|P_{23}|$, where the two limits can be clearly distinguished. (Recall that for slepton masses of $\sim 400\text{GeV}$ and $\tan\beta = 10$, $|P_{13,23}| < 0.09$, however, we show the results for $|P_{13,23}| < 1$ to allow for larger slepton masses or smaller values of $\tan\beta$.) In these figures, we have used the central values for the measured masses and mixing angles in [16], namely $m_2 = 8.9 \times 10^{-3}\text{eV}$, $m_3 = 4.7 \times 10^{-2}\text{eV}$, $\sin^2\theta_{12} = 0.30$ and $\sin^2\theta_{23} = 0.50$. These parameters have to be run from the electroweak scale to the decoupling scale [51] and this introduces corrections smaller than a 60% on the reconstructed parameters. In addition, the experimental error on the low energy parameters introduces an indeterminacy on the reconstructed parameters smaller than a factor of two, that will be reduced in forthcoming experiments. It is apparent from this analysis that the observation of rare decays would be an important step towards reconstructing the complete Lagrangian of the 2RHN model.

4.2 Complex case, $\theta_{13} = 0$

For the complex case, we will show first the results for $\theta_{13} = 0$, so that the high-energy parameters will depend only on the Majorana phase ϕ (recall that in this case P is real with a good approximation). Later on, we will discuss the situation with non-vanishing θ_{13}. On the other hand, for the atmospheric and solar angles, we maintain the experimentally favoured values $\theta_{23} \simeq \pi/4$, $\theta_{12} \simeq \pi/6$. The analytical expressions for the high energy parameters are given in this case by:
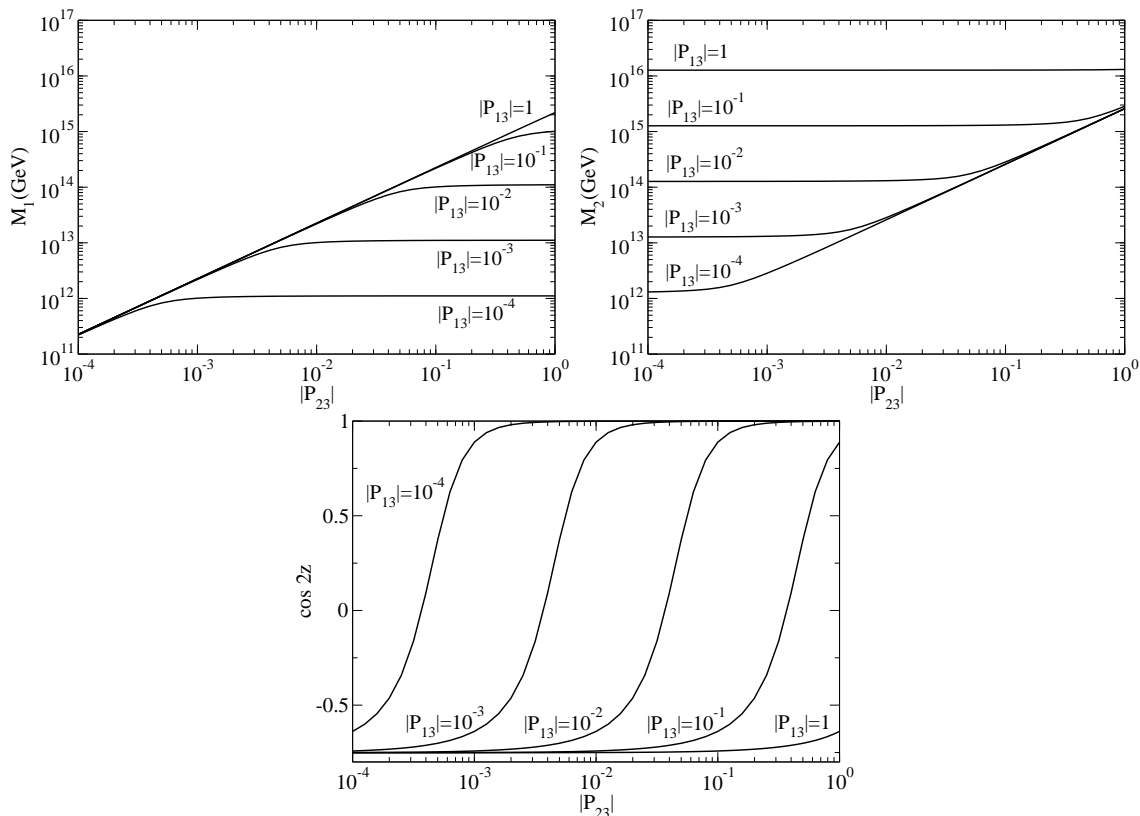
Figure 1: Reconstructed right-handed neutrino masses and parameter z in the matrix R, for different values of $|P_{13}|$ and $|P_{23}|$, in the limit $|P_{12}| \ll |P_{13}|, |P_{23}|$. Here, we have taken $\theta_{23} \simeq \pi/4$, $\theta_{12} \simeq \pi/6$, $\theta_{13} \simeq 0$ and $\phi \simeq 0$ (so that all the parameters involved in the reconstruction procedure are real to a good approximation).

- $|P_{13}| \ll |P_{23}|$

$$M_1 \simeq 2\sqrt{\frac{2}{3}} \frac{|P_{13}|}{m_2} \langle H_u^0 \rangle^2,$$

$$M_2 \simeq \frac{2|P_{23}|}{m_3} \langle H_u^0 \rangle^2,$$

$$\cos 2z \simeq 1. \tag{4.16}$$

Therefore, the Yukawa coupling is:

$$\mathbf{Y}_\nu \simeq \sqrt{|P_{23}|} \begin{pmatrix} \sqrt{\frac{|P_{13}|}{\sqrt{6}|P_{23}|}} e^{i\phi/2} & \sqrt{\sqrt{\frac{3}{8}} \frac{|P_{13}|}{|P_{23}|}} e^{i\phi/2} & -\sqrt{\sqrt{\frac{3}{8}} \frac{|P_{13}|}{|P_{23}|}} e^{i\phi/2} \\ -\frac{|P_{13}|}{2|P_{23}|} & 1 & 1 \end{pmatrix}. \tag{4.17}$$

Note that although in this limit the right-handed masses and $\cos 2z$ do not depend on the Majorana phase, the Yukawa coupling does. The dependence on the phase results from the phase in the leptonic mixing matrix, U, that enters in the parametrization of the Yukawa coupling, eq. (2.1). Similarly to the real case, the lightest right-handed mass is essentially determined by the rate for the process $\tau \to e\gamma$, while the heaviest one, by the process $\tau \to \mu\gamma$.

- $|P_{23}| \ll |P_{13}|$

$$M_1 \simeq \frac{8|P_{23}|\langle H_u^0\rangle^2}{\sqrt{9m_2^2 + 16m_3^2 + 24m_2 m_3 \cos\phi}},$$

$$M_2 \simeq \frac{|P_{13}|\sqrt{9m_2^2 + 16m_3^2 + 24m_2 m_3 \cos\phi}}{\sqrt{6}m_2 m_3}\langle H_u^0\rangle^2,$$

$$\cos 2z \simeq \frac{3m_2 e^{i\phi} - 4m_3}{3m_2 e^{i\phi} + 4m_3}, \tag{4.18}$$

so that the reconstructed Yukawa coupling is:

$$\mathbf{Y}_\nu \simeq \sqrt{\sqrt{6}\frac{\Delta}{\Delta^*}|P_{13}|} \left(\begin{matrix} \sqrt{\frac{\sqrt{6}|P_{23}|}{|P_{13}|}}\frac{m_2 e^{i\phi}}{|\Delta|^2} & \sqrt{\frac{|P_{23}|}{\sqrt{6}|P_{13}|}}\frac{\Delta^*}{\Delta} & \sqrt{\frac{|P_{23}|}{\sqrt{6}|P_{13}|}}\frac{-3m_2 e^{i\phi}+4m_3}{|\Delta|^2} \\ -\frac{e^{i\phi/2}}{\sqrt{6}} & \frac{\sqrt{6}m_2 e^{-i\phi/2}}{\Delta^2}\frac{|P_{23}|}{|P_{13}|} & e^{i\phi/2} \end{matrix} \right), \tag{4.19}$$

where $\Delta = \sqrt{3m_2 e^{-i\phi} + 4m_3}$. Contrary to the previous limit, here the right-handed masses and R do depend on the Majorana phase. Concerning which processes are relevant to determine which parameter, the conclusion is analogous to the real case: the lightest right-handed mass is essentially determined by the process $\tau \to \mu\gamma$, and the heaviest by $\tau \to e\gamma$.

The numerical results can be found in figure 2. We see that when $|P_{13}| \ll |P_{23}|$ the correlation between the right-handed masses and $|P_{13}|$ or $|P_{23}|$ is very tight, allowing a very precise reconstruction of the high-energy parameters. However, when $|P_{23}| \ll |P_{13}|$ the reconstruction is more complicated, and the precise determination of the high-energy parameters would require the measurement of the Majorana phase. In figure 2 we have sampled ϕ between 0 and 2π and show the regions at 2σ from the main value. These regions are fairly narrow, so even without knowing the Majorana phase, it could be possible to reconstruct the right-handed masses from rare decays, up to a factor of two.

On the other hand, for the numerical value of $\cos 2z$ we show both the absolute value and the argument as a function of the Majorana phase, ϕ, for fixed $|P_{23}| = 10^{-2}$ and for different values of $|P_{13}|$. It can be checked that the prediction for $\cos 2z$ depends on the ratio $|P_{13}|/|P_{23}|$, so the results for other values of $|P_{23}|$ could be read easily from this figure. We find that when $|P_{13}| \ll |P_{23}|$ both the absolute value and the argument of $\cos 2z$ are not very sensitive to ϕ. However, despite the matrix R depends weakly on ϕ, it is not possible to reconstruct the Yukawa coupling, due to the dependence of the Yukawa matrix with ϕ through the leptonic mixing matrix in eq. (2.1). On the other hand, when $|P_{23}| \ll |P_{13}|$, the absolute value of $\cos 2z$ depends strongly on ϕ. Therefore, in both cases the reconstruction of the Yukawa coupling would require the determination of the Majorana phase, although it would not be necessary for a rough reconstruction of the right-handed masses.

4.3 Complex case, $\theta_{13} \neq 0$

Finally we show the results including the effects of θ_{13} and the phase δ. Although it is possible to derive analytic expressions for the high-energy parameters in the different limits, the expressions are very complicated and difficult to analyze. Therefore, in this subsection we will limit ourselves to show the numerical results, that can be found in figure 3.
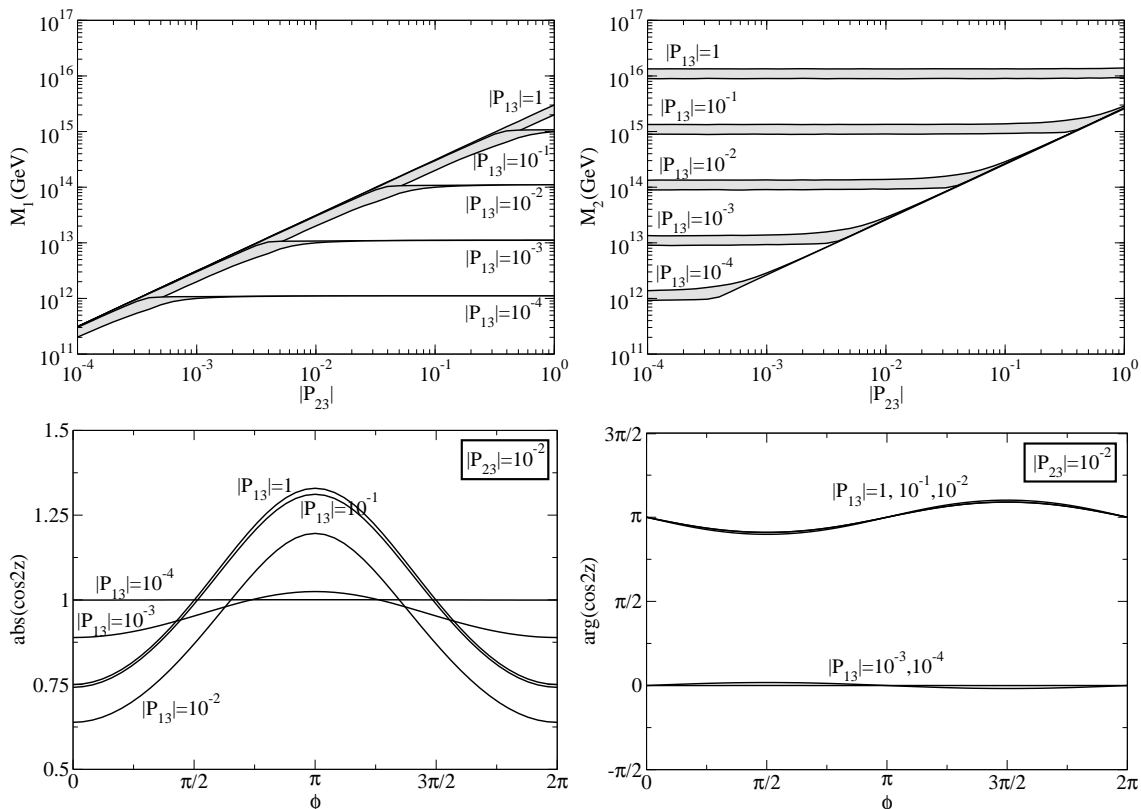
Figure 2: The same as figure 1, but sampling over different values for ϕ. The shaded areas represent the regions at 2σ from the main value. For $\cos 2z$ we plot both the absolute value and the argument as a function of the Majorana phase ϕ, for fixed $|P_{23}| = 10^{-2}$ and different values of $|P_{13}|$.

We find that in the whole parameter space, the measurement of θ_{13}, δ and ϕ would be desirable for a precise determination of the right-handed masses. However, even without knowing these parameters, the determination of the rates for the rare lepton decays would allow the reconstruction of the right-handed masses up to a factor of three.

On the other hand, when θ_{13} is large, the reconstruction of the parameter z necessarily requires the measurement of all the low-energy parameters. The situation is particularly critical when $|P_{13}| \ll |P_{23}|$, since $\cos 2z$ is very sensitive to the Dirac phase (when θ_{13} is large). For instance, when $\theta_{13} = 0.1$, the absolute value of $\cos 2z$ can vary between 0.6 and 1.5 when δ and ϕ vary between 0 and 2π. When $|P_{23}| \ll |P_{13}|$, the dispersion produced by the angle θ_{13} is smaller, between 0.8 and 1.3. Concerning the argument of $\cos 2z$ the range of values is smaller, although still important, particularly in the limit $|P_{13}| \ll |P_{23}|$. Therefore, when θ_{13} is large, it seems unavoidable the precise measurement of θ_{13}, δ and ϕ to determine $\cos 2z$ and to reconstruct the Yukawa coupling.

5. The 2RHN model as a limit of the 3RHN model

Although the 2RHN model can explain all the neutrino experiments, the 3RHN model is without any doubt much more appealing from the theoretical point of view. Nevertheless,
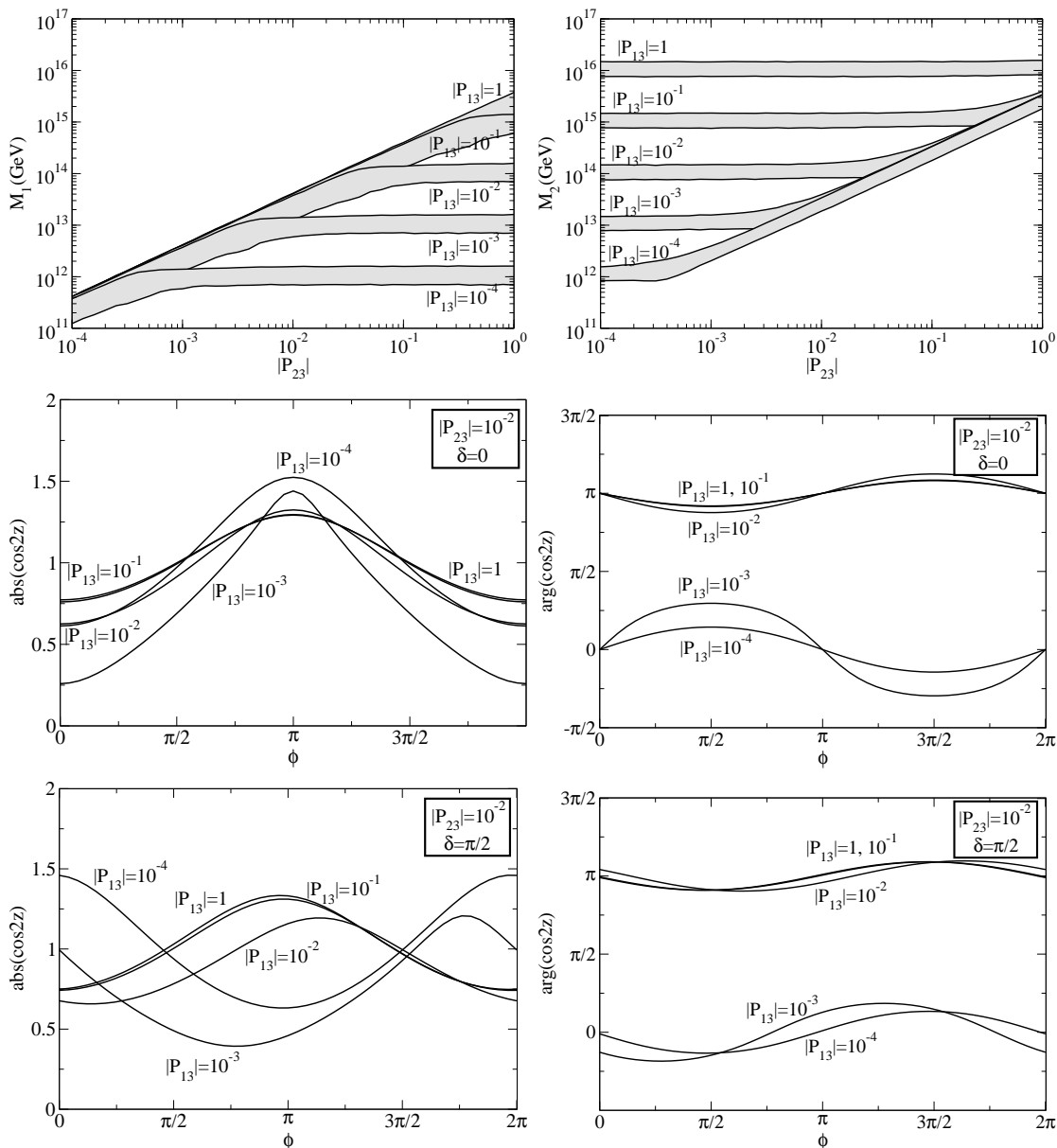
Figure 3: The same as figure 1, but sampling over different values for ϕ and δ for $\theta_{13} = 0.1$. The shaded areas represent the regions at 2σ from the main value. For $\cos 2z$ we plot both the absolute value and the argument as a function of the Majorana phase ϕ, for fixed $|P_{23}| = 10^{-2}$ and different values of $|P_{13}|$, and for two values of the phase δ: 0 and $\pi/2$.

there are some situations where the 3RHN model can be well approximated by a 2RHN model, both from the point of view of neutrino masses, as from the point of view of radiative corrections, parametrized by the matrix P. In these situations, the procedure presented in this paper to reconstruct the high-energy parameters would apply.

Let us discuss first the case of the neutrino mass matrix, that in the basis where the right-handed neutrino mass matrix is diagonal reads:

$$\mathcal{M}_{ij} = \frac{y_{1i}y_{1j}}{M_1} + \frac{y_{2i}y_{2j}}{M_2} + \frac{y_{3i}y_{3j}}{M_3}, \tag{5.1}$$

where $y_{ij} = (\mathbf{Y}_\nu)_{ij}$. Two right-handed neutrinos dominate the see-saw when

$$\frac{y_{1i}y_{1j}}{M_1} \ll \frac{y_{2i}y_{2j}}{M_2}, \frac{y_{3i}y_{3j}}{M_3} \quad \text{or}$$
$$\frac{y_{2i}y_{2j}}{M_2} \ll \frac{y_{1i}y_{1j}}{M_1}, \frac{y_{3i}y_{3j}}{M_3} \quad \text{or}$$
$$\frac{y_{3i}y_{3j}}{M_3} \ll \frac{y_{1i}y_{1j}}{M_1}, \frac{y_{2i}y_{2j}}{M_2} \quad \text{for all } i,j = 1,2,3. \tag{5.2}$$

The most interesting cases are the first and the third. The first one corresponds to the case in which the Yukawa couplings for the first generation of right handed neutrinos are tiny, $y_{1i} \ll y_{2i}, y_{3i}$, for $i = 1, 2, 3$. If this is the case, the radiative corrections are also dominated by the same two right-handed neutrinos, the two heaviest ones. Therefore, in this case the 3RHN model can be well approximated by a 2RHN model, both from the point of view of neutrino masses as of radiative corrections. Since the two relevant right-handed neutrinos are the two heaviest ones, the corresponding Yukawa couplings could be large, and the radiative corrections could be sizable.[4]

The third case corresponds to the situation where the mass of the heaviest right-handed neutrino is much larger than the mass of the other two, $M_3 \gg M_1, M_2$ [12]. However, in general the heaviest right-handed neutrino will produce sizable contributions to the radiative corrections. If this is the case, the 3RHN model could be reduced to a 2RHN model only from the point of view of neutrino masses, but not from the point of view of the radiative corrections. Nevertheless, there are some circumstances in which the heaviest right-handed neutrino indeed does not contribute to the radiative corrections and does not leave any imprint in P, so that this matrix is only determined by the Yukawa couplings of the two lightest generations of right-handed neutrinos. If this occurs, the 3RHN model would also be well approximated by a 2RHN model from the point of view of the radiative corrections. This situation arises for example when the mass of the heaviest right-handed neutrino is very close to the Planck mass, although this possibility seems a bit contrived.

A more plausible situation arises in models with gauge mediated supersymmetry breaking [52]. So far, we have implicitly assumed that the boundary conditions for the soft breaking terms are set at the Planck scale. However, if the mass of the messenger particles involved in the supersymmetry breaking mechanism is smaller than M_3 but larger than M_2, then the heaviest right-handed neutrino would decouple at an energy larger than the energy at which supersymmetry breaking is communicated to the observable sector. Consequently, it would not participate in the radiative corrections of the parameters of the Lagrangian. If this is the case, only the two lightest right-handed neutrinos would contribute to the radiative corrections and to the neutrino mass generation, and therefore the 3RHN model could be well approximated by a 2RHN model.

The experimental signature of this scenario would be a light gravitino, although probably not ultra-light, since the mass of the messenger has to be larger than the mass of the next-to-lightest right-handed neutrino, that is expected to be rather large. The gravitino

---

[4]In this scenario there would be no relation whatsoever between leptogenesis and low energy observables: leptogenesis would depend on the couplings of the lightest right-handed neutrino, whereas neutrino and slepton parameters would be determined by the couplings of the two heaviest right-handed neutrinos.

mass in these scenarios can be estimated as [53]

$$m_{3/2} = \frac{F}{\sqrt{3}M_P} \sim \frac{\pi}{\sqrt{3}\alpha} \frac{M_{\mathrm{mes}}}{M_P} \, \widetilde{m},$$ (5.3)

where we have assumed vanishing cosmological constant and $M_P = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18}\mathrm{GeV}$ is the reduced Planck mass. In this formula, F measures supersymmetry breaking in the messenger sector, M_{mes} is the mass of the messenger particles and $\widetilde{m} \sim (\alpha/\pi)F/M_{\mathrm{mes}}$ is the typical soft mass scale (recall that in gauge mediated supersymmetry breaking scenarios gaugino masses are generated at one loop and scalar masses at two loops). If only the heaviest right-handed neutrino decouples from the radiative corrections, it has to happen that $M_2 < M_{\mathrm{mes}} < M_3$. So, if the gravitino mass is measured, a lower bound on M_3 would follow:

$$M_3 \gtrsim \frac{\sqrt{3}\alpha}{\pi} \frac{m_{3/2}}{\widetilde{m}} M_P.$$ (5.4)

Alternatively, if M_2 can be reconstructed from low energy data, a lower bound on the gravitino mass would follow. Note that the precise determination of M_2 requires the knowledge of the cut-off scale, that is not known a priori. However one could compute M_2 assuming that the cut-off is set at the Planck scale instead of the actual messenger scale. Then, the value obtained for M_2 would be smaller (see the appendix for details) and therefore the bound on the gravitino mass obtained in this way would hold, although it would be more conservative than the actual bound.

Besides, when supersymmetry breaking is mediated to the observable sector through gauge interactions by particles with a mass smaller than the Planck mass, the soft terms are almost proportional to the identity at the cut-off scale. Thus, any flavour changing effect or mass splitting between sleptons or sneutrinos observed at low energies would be entirely due to radiative corrections. On the other hand, CP violating effects could originate in other sectors, such as the neutralino or the chargino sector, although there could be a contribution from P that might be disentangled.

Finally, if in addition the gravitino is the lightest supersymmetric particle, some interesting cosmological consequences would follow. For instance, the strong bounds on the reheating temperature, $T_R \lesssim 10^6\mathrm{GeV}$ for $m_{3/2} \sim \mathcal{O}(1\mathrm{TeV})$ [54], would be relaxed so that it could be as large as $10^{11}\mathrm{GeV}$ [55]. This has crucial consequences for the leptogenesis mechanism, since the mass of the lightest right-handed could be compatible with the constraints on the reheating temperature from preventing gravitino overproduction [47, 56].

6. Conclusions

The two right-handed neutrino model can explain all the neutrino oscillation experiments, but depends on less parameters than the conventional three right-handed neutrino model. Therefore, the serious problem of the lack of predictivity of the conventional see-saw model is softened. In this paper we have exploited this observation to argue that the high-energy parameters of the two right-handed neutrino model could be reconstructed using just low

energy experiments, provided supersymmetry is discovered and some hypotheses are made about the structure of the soft terms at the cut-off scale.

To this end, we have proposed an alternative parametrization of the two right-handed neutrino model just in terms of low energy observables, namely the neutrino mass matrix and the off-diagonal elements of the matrix $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$, that is responsible for the radiative corrections of the slepton parameters, and that in particular induces rare leptonic decays. We have discussed the present information available on these parameters and the prospects to improve our knowledge of them in the next few years. Except for the case of the Majorana phase, we find the prospects very encouraging.

We have presented an exact procedure that allows to reconstruct the high-energy superpotential of the two right-handed neutrino model in terms of the low energy parameters. We have applied this procedure to a particular scenario with $BR(\mu \to e\gamma) \ll BR(\tau \to (\mu, e)\gamma)$, and we have found that in the case where all the parameters are real and θ_{13} is small, the detection of the rare decays $\tau \to \mu\gamma$ and $\tau \to e\gamma$ would be a very important step towards reconstructing the high-energy parameters. We have also analyzed the impact of the phases in the leptonic mixing matrix on this conclusion, since they could be the worst determined among all the low energy parameters. We have found that when the angle θ_{13} is large and the phases in the leptonic mixing matrix do not vanish, the connection is more diffuse, although it could still be possible to determine the right-handed masses up to a factor of three. The reconstruction of the neutrino Yukawa coupling is more complicated, since in general would require the measurement of θ_{13}, δ and ϕ.

Finally, we have argued that this procedure does not apply only to the strict two right-handed neutrino model. There are limits of the three right-handed neutrino model that resemble a two right-handed neutrino model to a good approximation, both from the point of view of neutrino masses, as from the point of view of radiative corrections. In these limits, the procedure proposed in this paper applies.

Acknowledgments

A. An improved reconstruction procedure

Throughout the paper we have used as parameters to reconstruct the high-energy theory $P = \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ and $\mathcal{M} = \mathbf{Y}_\nu^T \text{diag}(M_1^{-1}, M_2^{-1}) \mathbf{Y}_\nu \langle H_u^0 \rangle^2$. However, to disentangle P it is necessary to know not only the cut-off scale but also the decoupling scales, that are not known a priori. Nevertheless, the procedure can be applied recursively in order to reconstruct the high-energy parameters.

In this appendix it will be shown that this recursive procedure could be avoided using as low energy parameters of the 2RHN model

$$\mathcal{M} = \mathbf{Y}_\nu^T \text{diag}(M_1^{-1}, M_2^{-1}) \mathbf{Y}_\nu \langle H_u^0 \rangle^2,$$

$$P = \mathbf{Y}_\nu^\dagger \text{diag}\left(\log\frac{\Lambda}{M_1}, \log\frac{\Lambda}{M_2}\right)\mathbf{Y}_\nu, \tag{A.1}$$

where the second parameter is the combination that appears in the leading-log approximation to the solution of the renormalization group equations, and Λ is the cut-off scale.

Defining the new parameters

$$\widetilde{\mathbf{Y}}_\nu = \text{diag}\left(\sqrt{\log\frac{\Lambda}{M_1}}, \sqrt{\log\frac{\Lambda}{M_2}}\right)\mathbf{Y}_\nu,$$

$$\widetilde{M}_1 = M_1 \log\frac{\Lambda}{M_1},$$

$$\widetilde{M}_2 = M_2 \log\frac{\Lambda}{M_1} \tag{A.2}$$

and substituting in eq. (A.1), one finds that

$$\mathcal{M} = \widetilde{\mathbf{Y}}_\nu^T \text{diag}(\widetilde{M}_1^{-1}, \widetilde{M}_2^{-1})\widetilde{\mathbf{Y}}_\nu \langle H_u^0\rangle^2,$$

$$P = \widetilde{\mathbf{Y}}_\nu^\dagger \widetilde{\mathbf{Y}}_\nu. \tag{A.3}$$

Therefore, we can apply the general formulas eq. (4.1) to solve for the parameters $\widetilde{\mathbf{Y}}_\nu$, $\widetilde{M}_1$ and $\widetilde{M}_2$. Finally, inverting eq. (A.2) we can reconstruct the actual high-energy parameters $\mathbf{Y}_\nu$, M_1 and M_2. Note that there are always two solutions for M_1 and M_2, one larger than Λ/e and the other smaller.

From eq. (A.2) it is possible to estimate the impact of our ignorance of the cut-off scale on the reconstructed values of the right-handed masses. From the reconstruction procedure it is always possible to compute $\widetilde{M}_1$ and $\widetilde{M}_2$, and from eq. (A.2), the actual masses M_1 and M_2, that would depend on the cut-off scale. If we set the cut-off scale at the Planck mass, we would obtain $M_1(M_P)$ and $M_2(M_P)$. However, it could happen that the actual cut-off scale is smaller than the Planck mass, for instance in models with gauge mediated supersymmetry breaking; in that case we would obtain $M_1(\Lambda)$ and $M_2(\Lambda)$. In figure 4 we compare the values of the masses computed assuming that the cut-off is set at the Planck mass or at a different cut-off, for fixed $\widetilde{M}_1$ or $\widetilde{M}_2$. We find that taking as the cut-off the Planck mass instead of an intermediate scale underestimates the value of the reconstructed right-handed masses. Nevertheless, the error made is usually smaller than one order of magnitude.

References

[1] P. Minkowski, $\mu \to e\gamma$ *at a rate of one out of 1-billion muon decays?*, *Phys. Lett.* **B 67** (1977) 421;
M. Gell-Mann, P. Ramond and R. Slansky, *Proceedings of the supergravity Stony Brook workshop*, New York 1979, P. Van Nieuwenhuizen and D. Freedman eds.,
T. Yanagida, *Proceedinds of the workshop on unified theories and baryon number in the universe*, Tsukuba, Japan 1979, A. Sawada and A. Sugamoto eds.,
R.N. Mohapatra and G. Senjanovic, *Neutrino mass and spontaneous parity nonconservation*, *Phys. Rev. Lett.* **44** (1980) 912; *Neutrino masses and mixings in gauge models with spontaneous parity violation*, *Phys. Rev.* **D 23** (1981) 165;
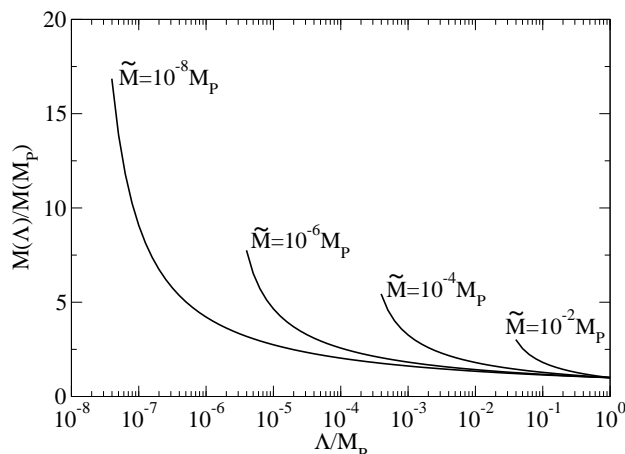
Figure 4: Relation between the right-handed neutrino mass reconstructed using the actual cut-off scale Λ, and reconstructed using the Planck mass as cut-off, for different values of $\widetilde{M}$ (see details in the appendix).

S.L. Glashow, *The future of elementary particle physics*, in *Cargese 1979, proceedings, quarks and leptons*, pp. 687-713 Harvard University, Cambridge
J. Schechter and J.W.F. Valle, *Neutrino masses in* SU(2) × U(1) *theories*, *Phys. Rev.* **D 22** (1980) 2227.

[2] A. Broncano, M.B. Gavela and E. Jenkins, *The effective lagrangian for the seesaw model of neutrino mass and leptogenesis*, *Phys. Lett.* **B 552** (2003) 177 [hep-ph/0210271].

[3] F. Borzumati and A. Masiero, *Large muon and electron number violations in supergravity theories*, *Phys. Rev. Lett.* **57** (1986) 961.

[4] J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi and T. Yanagida, *Lepton flavor violation in the supersymmetric standard model with seesaw induced neutrino masses*, *Phys. Lett.* **B 357** (1995) 579 [hep-ph/9501407];
J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, *Lepton-flavor violation via right-handed neutrino yukawa couplings in supersymmetric standard model*, *Phys. Rev.* **D 53** (1996) 2442 [hep-ph/9510309].

[5] J.A. Casas and A. Ibarra, *Oscillating neutrinos and* $\mu \to e, \gamma$, *Nucl. Phys.* **B 618** (2001) 171 [hep-ph/0103065].

[6] S. Lavignac, I. Masina and C.A. Savoy, $\tau \to \mu\gamma$ *and* $\mu \to e\gamma$ *as probes of neutrino mass models*, *Phys. Lett.* **B 520** (2001) 269 [hep-ph/0106245]; *Large solar angle and seesaw mechanism: a bottom-up perspective*, *Nucl. Phys.* **B 633** (2002) 139 [hep-ph/0202086].

[7] A. Romanino and A. Strumia, *Electron and muon electric dipoles in supersymmetric scenarios*, *Nucl. Phys.* **B 622** (2002) 73 [hep-ph/0108275];
J.R. Ellis, J. Hisano, S. Lola and M. Raidal, *CP-violation in the minimal supersymmetric seesaw model*, *Nucl. Phys.* **B 621** (2002) 208 [hep-ph/0109125].

[8] H. Baer, C. Balazs, J.K. Mizukoshi and X. Tata, *Can precision measurements of slepton masses probe righthanded neutrinos?*, *Phys. Rev.* **D 63** (2001) 055011 [hep-ph/0010068];
G.A. Blair, W. Porod and P.M. Zerwas, *The reconstruction of supersymmetric theories at high energy scales*, *Eur. Phys. J.* **C 27** (2003) 263 [hep-ph/0210058];
A. Freitas, W. Porod and P.M. Zerwas, *Determining sneutrino masses and physical implications*, *Phys. Rev.* **D 72** (2005) 115002 [hep-ph/0509056].

[9] S. Davidson and A. Ibarra, *Determining seesaw parameters from weak scale measurements?*, *JHEP* **09** (2001) 013 [`hep-ph/0104076`].

[10] J.R. Ellis, J. Hisano, M. Raidal and Y. Shimizu, *A new parametrization of the seesaw mechanism and applications in supersymmetric models*, *Phys. Rev.* **D 66** (2002) 115013 [`hep-ph/0206110`].

[11] A. Kleppe, *Extending the Standard Model with two right-handed neutrinos*, in *Lohusalu 1995, Neutrino physics*, pp. 118–125; *Extending the Standard Model by including right-handed neutrinos*, prepared for *Workshop on what comes beyond the Standard Model*, Bled, Slovenia, 29 Jun - 9 Jul 1998;
E. Ma, D.P. Roy and U. Sarkar, *A see-saw model for atmospheric and solar neutrino oscillations*, *Phys. Lett.* **B 444** (1998) 391 [`hep-ph/9810309`];
P.H. Frampton, S.L. Glashow and T. Yanagida, *Cosmological sign of neutrino CP-violation*, *Phys. Lett.* **B 548** (2002) 119 [`hep-ph/0208157`];
M. Raidal and A. Strumia, *Predictions of the most minimal see-saw model*, *Phys. Lett.* **B 553** (2003) 72 [`hep-ph/0210021`];
V. Barger, D.A. Dicus, H.-J. He and T.-j. Li, *Structure of cosmological CP-violation via neutrino seesaw*, *Phys. Lett.* **B 583** (2004) 173 [`hep-ph/0310278`];
W.-L. Guo and Z.-z. Xing, *Calculable CP-violating phases in the minimal seesaw model of leptogenesis and neutrino mixing*, *Phys. Lett.* **B 583** (2004) 163 [`hep-ph/0310326`];
R. Gonzalez Felipe, F.R. Joaquim and B.M. Nobre, *Radiatively induced leptogenesis in a minimal seesaw model*, *Phys. Rev.* **D 70** (2004) 085009 [`hep-ph/0311029`];
S. Raby, *A natural framework for bi-large neutrino mixing*, *Phys. Lett.* **B 561** (2003) 119 [`hep-ph/0302027`];
B. Dutta and R.N. Mohapatra, *Lepton flavor violation and neutrino mixings in a 3x2 seesaw model*, *Phys. Rev.* **D 68** (2003) 056006 [`hep-ph/0305059`];
R. Kuchimanchi and R.N. Mohapatra, *Testing leptonic* SU(2) *horizontal symmetry using neutrino mixings*, *Phys. Lett.* **B 552** (2003) 198 [`hep-ph/0207373`];
T. Appelquist and R. Shrock, *Neutrino masses in theories with dynamical electroweak symmetry breaking*, *Phys. Lett.* **B 548** (2002) 204 [`hep-ph/0204141`];
T. Appelquist and R. Shrock, *Dynamical symmetry breaking of extended gauge symmetries*, *Phys. Rev. Lett.* **90** (2003) 201801 [`hep-ph/0301108`];
T. Endoh, S. Kaneko, S.K. Kang, T. Morozumi and M. Tanimoto, *CP violation in neutrino oscillation and leptogenesis*, *Phys. Rev. Lett.* **89** (2002) 231601 [`hep-ph/0209020`];
S. Chang, S.K. Kang and K. Siyeon, *Minimal seesaw model with tri/bi-maximal mixing and leptogenesis*, *Phys. Lett.* **B 597** (2004) 78 [`hep-ph/0404187`].

[12] A.Y. Smirnov, *Seesaw enhancement of lepton mixing*, *Phys. Rev.* **D 48** (1993) 3264 [`hep-ph/9304205`];
M. Jezabek and Y. Sumino, *Neutrino mixing and seesaw mechanism*, *Phys. Lett.* **B 440** (1998) 327 [`hep-ph/9807310`];
S.F. King, *Atmospheric and solar neutrinos from single right-handed neutrino dominance and* U(1) *family symmetry*, *Nucl. Phys.* **B 562** (1999) 57 [`hep-ph/9904210`]; *Large mixing angle MSW and atmospheric neutrinos from single right-handed neutrino dominance and* U(1) *family symmetry*, *Nucl. Phys.* **B 576** (2000) 85 [`hep-ph/9912492`]; *Constructing the large mixing angle MNS matrix in see-saw models with right-handed neutrino dominance*, *JHEP* **09** (2002) 011 [`hep-ph/0204360`];
S.F. King and G.G. Ross, *Fermion masses and mixing angles from* SU(3) *family symmetry and unification*, *Phys. Lett.* **B 574** (2003) 239 [`hep-ph/0307190`];

W. Rodejohann, *Hierarchical matrices in the see-saw mechanism, large neutrino mixing and leptogenesis*, *Eur. Phys. J.* **C 32** (2004) 235 [`hep-ph/0311142`];
H.K. Dreiner, H. Murayama and M. Thormeier, *Anomalous flavor* U(1)$_X$ *for everything*, *Nucl. Phys.* **B 729** (2005) 278 [`hep-ph/0312012`].

[13] Z. Maki, M. Nakagawa and S. Sakata, *Remarks on the unified model of elementary particles*, *Prog. Theor. Phys.* **28** (1962) 870.

[14] A. Ibarra and G.G. Ross, *Neutrino properties from Yukawa structure*, *Phys. Lett.* **B 575** (2003) 279 [`hep-ph/0307051`].

[15] A. Ibarra and G.G. Ross, *Neutrino phenomenology: the case of two right handed neutrinos*, *Phys. Lett.* **B 591** (2004) 285 [`hep-ph/0312138`].

[16] M. Maltoni, T. Schwetz, M.A. Tortola and J.W.F. Valle, *Status of global fits to neutrino oscillations*, *New J. Phys.* **6** (2004) 122 [`hep-ph/0405172`].

[17] ICARUS collaboration, P. Aprili et al., *The ICARUS experiment: a second-generation proton decay experiment and neutrino observatory at the Gran Sasso laboratory. Cloning of T600 modules to reach the design sensitive mass. (Addendum)*, CERN-SPSC-2002-027.

[18] OPERA collaboration, M. Guler et al., *OPERA: an appearance experiment to search for $\nu_\mu \leftrightarrow \nu_\tau$ oscillations in the CNGS beam. Experimental proposal*, CERN-SPSC-2000-028.

[19] MINOS collaboration, E. Ables et al., *P-875: a Long baseline neutrino oscillation experiment at Fermilab*, FERMILAB-PROPOSAL-0875.

[20] NOvA collaboration, D.S. Ayres et al., *Nova proposal to build a 30-kiloton off-axis detector to study neutrino oscillations in the Fermilab numi beamline*, `hep-ex/0503053`.

[21] Y. Itow et al., *The JHF-Kamioka neutrino project*, `hep-ex/0106019`.

[22] F. Ardellier et al., *Letter of intent for double-Chooz: a search for the mixing angle θ_{13}*, `hep-ex/0405032`.

[23] S. Choubey and S.T. Petcov, *Reactor anti-neutrino oscillations and gadolinium loaded super-Kamiokande detector*, *Phys. Lett.* **B 594** (2004) 333 [`hep-ph/0404103`].

[24] A. Bandyopadhyay, S. Choubey, S. Goswami and S.T. Petcov, *High precision measurements of θ_{solar} in solar and reactor neutrino experiments*, *Phys. Rev.* **D 72** (2005) 033013 [`hep-ph/0410283`].

[25] P. Hernandez, *Towards precision lepton flavour physics*, talk given at *What's ν, IFT MiniWorkshop on Neutrino Physics 2005*, Madrid, Spain, May 2005.

[26] S. Pascoli, S.T. Petcov and T. Schwetz, *The absolute neutrino mass scale, neutrino mass spectrum, majorana CP-violation and neutrinoless double-beta decay*, *Nucl. Phys.* **B 734** (2006) 24 [`hep-ph/0505226`].

[27] MEGA collaboration, M.L. Brooks et al., *New limit for the family-number non-conserving decay $\mu^+ \to e^+\gamma$*, *Phys. Rev. Lett.* **83** (1999) 1521 [`hep-ex/9905013`].

[28] BELLE collaboration, K. Abe et al., *An upper bound on the decay $\tau \to \mu\gamma$ from Belle*, *Phys. Rev. Lett.* **92** (2004) 171802 [`hep-ex/0310029`].

[29] BABAR collaboration, B. Aubert et al., *Search for lepton flavor violation in the decay $\tau \to \mu\gamma$*, *Phys. Rev. Lett.* **95** (2005) 041802 [`hep-ex/0502032`].

[30] K. Hayasaka et al., *Search for $\tau \to e\gamma$ decay at Belle*, *Phys. Lett.* **B 613** (2005) 20 [hep-ex/0501068].

[31] A. Masiero, S.K. Vempati and O. Vives, *Massive neutrinos and flavour violation*, *New J. Phys.* **6** (2004) 202 [hep-ph/0407325].

[32] A. Baldini, *Status of the MEG experiment*, *AIP Conf. Proc.* **721** (2004) 289.

[33] P. Wintz, in *Proceedings of the first international symposium on lepton and baryon number violation*, H.V. Klapdor-Kleingrothaus and I.V. Krivosheina, eds., Institute of Physics, Bristol/Philadelphia 1998, p. 534.

[34] A. van der Schaaf, *μe conversion experiments: status and prospects*, *Nucl. Instrum. Meth.* **A 503** (2003) 281.

[35] A. Sato, *PRIME PRISM muon to electron conversion experiment*, talk given at *NuFact05*, Frascati, Italy, June 2005.

[36] J. Aysto et al., *Physics with low-energy muons at a neutrino factory complex*, hep-ph/0109217.

[37] SuperKEKB Physics Working Group collaboration, A.G. Akeroyd et al., *Physics at super b factory*, hep-ex/0406071; T. Iijima, *Overview of physics at super B-factory*, talk given at the *6th Workshop on a higher luminosity B factory*, KEK, Tsukuba, Japan, November 2004.

[38] B.C. Regan, E.D. Commins, C.J. Schmidt and D. DeMille, *New limit on the electron electric dipole moment*, *Phys. Rev. Lett.* **88** (2002) 071805.

[39] D. Kawall, F. Bay, S. Bickman, Y. Jiang and D. DeMille, *Proof of principle for a high sensitivity search for the electric dipole moment of the electron using the metastable $a(1)[^3\Sigma^+]$ state of PbO*, *Phys. Rev. Lett.* **92** (2004) 133007 [hep-ex/0309079].

[40] D. Kawall, F. Bay, S. Bickman, Y. Jiang and D. DeMille, *Progress towards measuring the electric dipole moment of the electron in metastable PbO*, *AIP Conf. Proc.* **698** (2004) 192.

[41] S.K. Lamoreaux, *Solid state systems for electron electric dipole moment and other fundamental measurements*, nucl-ex/0109014.

[42] C.Y. Liu and S.K. Lamoreaux, *A new search for a permanent dipole moment of the electron in a solid state system*, *Mod. Phys. Lett.* **A 19** (2004) 1235.

[43] D.A. Demir and Y. Farzan, *Can measurements of electric dipole moments determine the seesaw parameters?*, *JHEP* **10** (2005) 068 [hep-ph/0508236].

[44] P.G. Harris et al., *New experimental limit on the electric dipole moment of the neutron*, *Phys. Rev. Lett.* **82** (1999) 904.

[45] Y.K. Semertzidis et al., *Sensitive search for a permanent muon electric dipole moment*, hep-ph/0012087.

[46] Belle collaboration, K. Inami et al., *Search for the electric dipole moment of the τ lepton*, *Phys. Lett.* **B 551** (2003) 16 [hep-ex/0210066].

[47] S. Davidson and A. Ibarra, *A lower bound on the right-handed neutrino mass from leptogenesis*, *Phys. Lett.* **B 535** (2002) 25 [hep-ph/0202239].

[48] P.H. Chankowski and K. Turzynski, *Limits on T_{RH} for thermal leptogenesis with hierarchical neutrino masses*, *Phys. Lett.* **B 570** (2003) 198 [hep-ph/0306059].

[49] M. Fukugita and T. Yanagida, *Baryogenesis without grand unification*, *Phys. Lett.* **B 174** (1986) 45.

[50] Previous attempts to reconstruct the high energy lagrangian from low energy observables can be found in refs. [2, 9, 10], as well as in S. Davidson, *From weak-scale observables to leptogenesis*, *JHEP* **03** (2003) 037 [hep-ph/0302075];
E.K. Akhmedov, M. Frigerio and A.Y. Smirnov, *Probing the seesaw mechanism with neutrino data and leptogenesis*, *JHEP* **09** (2003) 021 [hep-ph/0305322].

[51] K.S. Babu, C.N. Leung and J.T. Pantaleone, *Renormalization of the neutrino mass operator*, *Phys. Lett.* **B 319** (1993) 191 [hep-ph/9309223];
P.H. Chankowski and Z. Pluciennik, *Renormalization group equations for seesaw neutrino masses*, *Phys. Lett.* **B 316** (1993) 312 [hep-ph/9306333];
J.R. Ellis and S. Lola, *Can neutrinos be degenerate in mass?*, *Phys. Lett.* **B 458** (1999) 310 [hep-ph/9904279];
J.A. Casas, J.R. Espinosa, A. Ibarra and I. Navarro, *Nearly degenerate neutrinos, supersymmetry and radiative corrections*, *Nucl. Phys.* **B 569** (2000) 82 [hep-ph/9905381];
Naturalness of nearly degenerate neutrinos, *Nucl. Phys.* **B 556** (1999) 3 [hep-ph/9904395];
General RG equations for physical neutrino parameters and their phenomenological implications, *Nucl. Phys.* **B 573** (2000) 652 [hep-ph/9910420];
J.A. Casas, J.R. Espinosa, A. Ibarra and I. Navarro, *Naturalness of nearly degenerate neutrinos*, *Nucl. Phys.* **B 556** (1999) 3 [hep-ph/9904395];
J.A. Casas, J.R. Espinosa, A. Ibarra and I. Navarro, *General RG equations for physical neutrino parameters and their phenomenological implications*, *Nucl. Phys.* **B 573** (2000) 652 [hep-ph/9910420];
S. Antusch, J. Kersten, M. Lindner and M. Ratz, *Running neutrino masses, mixings and CP phases: analytical results and phenomenological consequences*, *Nucl. Phys.* **B 674** (2003) 401 [hep-ph/0305273];
S. Antusch, M. Drees, J. Kersten, M. Lindner and M. Ratz, *Neutrino mass operator renormalization revisited*, *Phys. Lett.* **B 519** (2001) 238 [hep-ph/0108005].

[52] L. Alvarez-Gaumé, M. Claudson and M.B. Wise, *Low-energy supersymmetry*, *Nucl. Phys.* **B 207** (1982) 96.

[53] G.F. Giudice and R. Rattazzi, *Theories with gauge-mediated supersymmetry breaking*, *Phys. Rept.* **322** (1999) 419 [hep-ph/9801271].

[54] M. Kawasaki, K. Kohri and T. Moroi, *Big-bang nucleosynthesis and hadronic decay of long-lived massive particles*, *Phys. Rev.* **D 71** (2005) 083502 [astro-ph/0408426];
M. Kawasaki, K. Kohri and T. Moroi, *Hadronic decay of late-decaying particles and big-bang nucleosynthesis*, *Phys. Lett.* **B 625** (2005) 7 [astro-ph/0402490].

[55] J.R. Ellis, D.V. Nanopoulos and S. Sarkar, *The cosmology of decaying gravitinos*, *Nucl. Phys.* **B 259** (1985) 175;
V.S. Berezinsky, *Cosmology of gravitino as the lightest supersymmetric particle*, *Phys. Lett.* **B 261** (1991) 71;
M. Bolz, W. Buchmuller and M. Plumacher, *Baryon asymmetry and dark matter*, *Phys. Lett.* **B 443** (1998) 209 [hep-ph/9809381].

[56] W. Buchmuller, P. Di Bari and M. Plumacher, *Leptogenesis for pedestrians*, *Ann. Phys. (NY)* **315** (2005) 305 [hep-ph/0401240];
G.F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, *Towards a complete theory of thermal leptogenesis in the SM and MSSM*, *Nucl. Phys.* **B 685** (2004) 89 [hep-ph/0310123].